End Sem(IV) —
Math (GE – 4)
Sec. A & B

2022

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer from any **one** Section.*

Section – A

(2019-22 Session)

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : 1×10 = 10

   (a) State the Fundamental theorem of integral calculus.

(b) Give an example of non-monotonic Riemann integrable function on [a, b].

(c) Write the Cauchy-Riemann equations.

(d) What do you mean by Analytic function ?

(e) Give an example of uncountable set.

(f) Define Idempotent matrix.

(g) Define set with an appropriate example.

(h) Define Symmetric Matrix.

(i) What do you mean by equivalence relation ?

(j) Define Involutory matrix.

2. Write $f(z) = z^3 + z + 1$ in the form of u + iv. 5

3. Prove that $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ is nilpotent. 5

Group – B

Answer any **four** questions of the following :

4. (a) If f is integrable on [a, b], then f^2 is also integrable. 10

(b) Prove that if f is bounded and integrable in [a, b], then there exists a number μ lying between the bounds of f such that

$$\int_a^b f(x)dx = \mu(b-a).$$ 10

5. (a) Explain Vandermonde and circulant matrix each with one example. 10

(b) Prove that two equivalence class are either equal or disjoint. 10

6. (a) State and prove De-Morgan's laws. 10

(b) Let R be a relation on the set of all integers $\mathbb{Z}$ defined by $x \equiv y \pmod 5$. Show that R is an equivalence relation on $\mathbb{Z}$. 10

7. (a) Show that $\lim_{z \to 0} \frac{z}{\bar{z}}$ does not exist. 10

(b) Show that $f(z) = |z|^2$ is nowhere analytic. 10

8. (a) Let $A = \{1, 2\}$, $B = \{x, y, z\}$ and $C = \{3, 4, 5\}$. Find $A \times B \times C$. 10

(b) Let A = {1, 2, 3; 4, 5, 6}, and let R be the relation on A defined by "x divides y", written x | y. Write R as the set of ordered pairs. Also, find R^{-1}. 10

9. (a) Let $f(z) = z^2$. Then, show that $f'(z) = 2z$. 10

(b) Show that the function $F(z) = e^x \cos y$ is harmonic and find its harmonic conjugate. 10

10. (a) Explain with an example when a system of linear equations has infinite solutions. 10

(b) Solve $\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. How many solution(s) is possible ? 10

Section – B

(2018-21 Session)

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : $1 \times 10 = 10$

(a) What are Fibonacci Numbers ?

(b) What are irreducible polynomials ?

(c) Give an example of positive Semi-definite matrix.

(d) Define Closure law.

(e) Define Linear code.

(f) Define Idempotent matrix.

(g) What is the odd parity of 111001 ?

(h) Define Symmetric matrix.

(i) Find the hamming distance between 111000 and 101010.

(j) Define Involuntary matrix.

2. Prove that the set of integers is a group under addition modulo 5. 5

3. Prove that $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ is nilpotent. 5

Group – B

Answer any **four** questions of the following :

4. (a) Explain permutation group. Give an example. 10

(b) Explain Vandermonde matrix with an example. 10

5. (a) Explain Hadamard matrix with an example. 10

(b) Explain circulant matrix with an example. 10

6. (a) Explain generating function. 10

(b) Explain how generating function is applied to count non-isomorphic graph. 10

7. (a) Give example of a set of code-word which is linear but not cyclic. 10

(b) Explain hamming detection and correction algorithm. 10

8. (a) Prepare a table for addition and multiplication for $\mathbb{F}_2[x] / (x^2 + x + 1)$. 10

(b) Prepare a table for (7, 4) code generated by $1 + x + x^3$. 10

9. (a) Explain forward and backward substitutions each with an example. 10

(b) Define LDU factorization with an example.

10

10. (a) Why we need an approximate solutions of system of linear equations. Explain it by considering an example of inconsistent system.

10

(b) Explain Normal equations with an example.

10